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Module 10 Packet

Numerical Expressions

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Meets with Excellence	Meets Proficiency	Developing Proficiency	Well Below Proficiency	Not Enough Evidence
You exceed the learning targets in understanding or application.	You have met all the learning targets for this standard.	You are approaching the standard or have only partial understanding.	You have not yet met many of components of the standard.	You haven't shown enough evidence for the standard.

Standard 6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.

tandard 6.EE.3 Apply the properties of operations to generate equivalent expressions.

Standard 6.EE.4 Identify when two expressions are equivalent.

In Module 10, you will learn how to...

- ____ 10-1 Writing and evaluating numerical expressions with exponents. (6.EE.1)
- ____10-2 Evaluating numerical expressions with exponents. (6.EE.1)
- ____10-3 Evaluating and comparing numerical expressions. (6.EE.1 and 6.EE.4)
- ____ 10-4 Writing equivalent expressions (6.EE.3)

Writing and Evaluating Numerical Expressions with Exponents

Learning Target: I can write and evaluate numerical expressions with exponents. (6.EE.1)

Do Now

Solve each expression. As you solve, pay attention to how you arrive at your answers

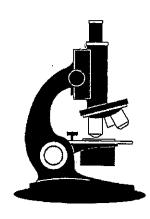
*You can add to evaluate each expression but multiplication is faster.

Example 1: Using Exponents

Multiplication is a faster way to solve expressions with repeated addition. When we add five groups of 10, we use an abbreviation and a notation, called multiplication.

A scientist observed the hourly growth of bacteria and recorded his observations in a table.

# Time (hours)	Total Bacteria
0	1
1	2
2	2 x 2 = 4
3	2 x 2 x 2 = 8
4	2 x 2 x 2 x 2 = 16



If multiplication is a more efficient way to represent addition problems involving the repeated addition of the same addend, do you think there might be a more efficient way to represent the repeated multiplication of the same factor, as in...

$$10 \times 10 \times 10 \times 10 \times 10 = ?$$

You can use an exponent and a base to write an expression with repeated multiplication.

Example: 7^3 means the product of three 7's.

$$7^{7} = 7 \times 7 \times 7$$

The base is the number that is multiplied, or the repeated factor.

The exponent tells how many times the base appears in the expression.

Expression	How to Read the Expression
6 ²	6 squared; 6 to the power of 2; 6 raised to the 2nd power
7 ³	7 cubed; 7 to the power of 3; 7 raised to the 3rd power
9 ⁴	9 to the power of 4; 9 raised to the 4th power

Writing Expressions in Exponential Form

A) 3x3x3x3x3

- Find the base, or the number being multiplied. The base is

- Find the exponent by counting the number of 3's being multiplied. The exponent is

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NAME OF THE PARTY	
$3 \times 3 \times 3 \times 3 \times 3 = \frac{3}{5 \text{ factors of } 3}$	5

5 to the power of 5 raised to the 5 power

Word Fo

B) $\frac{4}{5}$ x $\frac{4}{5}$ x $\frac{4}{5}$ x $\frac{4}{5}$

Find the base, or the number being multiplied. The base is ______

- Count the number of times $\frac{4}{5}$ appears in the expression. The exponent is $\underline{}$

Exponential Forn	Word Form
$\frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = -$	to the power of 4
$ \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \{4 \text{ factors of } \frac{4}{5}\} $	raised to the 4th power
	5

 Note: The base ⁴/₅ must be in parentheses to show that the entire fraction is used for repeated multiplication, not just the numerator.

Exercise 1

Write the following in wo	d form.		
1) 108	10 to the power	of 8; 10 raised to the	8th power
2) 3 x 3 x 3 x 3 x 3 z 3 5	3 to the power	of 5; 3 raised to the	5th power
3) 7 ²	•	the power of 2; 7 raised	
4) 1.9 4	1)	f 4; 1.9 raised to the 4#	
5) .(3/4) 5	3 to the power o	f5; = raised to the	5th power
Write each expression in	•	9	
1) 4 x 4 x 4 x 4 x 4 x 4 =	40	2) 6 = 6	
3) $\frac{1}{8} \times \frac{1}{8} = \left(\frac{1}{8} \right)^2$		4) 2.7 x 2.7 x 2.7 = 2.7 ³	<u>.</u>

Example 2- Evaluating Expressions (Finding the Value of Exponential Expressions)

To find the value of an expression, remember that the exponent indicates the number of times to use the base as a factor.

10⁴

- Identify the base and the exponent.
- The base is 10, and the exponent is 4

Evaluate the expression: $10^4 = \frac{10}{\text{Expanded Form}} \times \frac{10}{\text{Form}} \times \frac{10}{\text{Value}} = \frac{10,000}{\text{Value}}$

Exponential Expressio	n Expanded Form	Value
1 ³	1×1×1	· 1
$(\frac{2}{3})^2$	2 × 2 3	4 > 2
0.43	0.4 x 0.4 x 0.4	0.064
$(\frac{3}{5})^{0}$		1

Property of Zero as an Exponent

The value of any nonzero number raised to the power of 0 is 1.

Example: $5^0 = 1$

1. Complete the chart bel	low:
---------------------------	------

Exponential Form	Expanded Form	Value
34	3 x 3 x 3 x 3	81
20		1
$\left(\frac{2}{5}\right)^3$	号×号×号	16
19	/x/x/x/x/x/x/x/x/x/	9
5'	5	5
y ⁴ Remember, a variable is just a symbol for a number we don't know yet.	y.y.y.y	

2. What is the difference between 3g and g^3 ?

3×9 6

3×9×9

3. What is the value of a number raised to the power of 1?

3

12

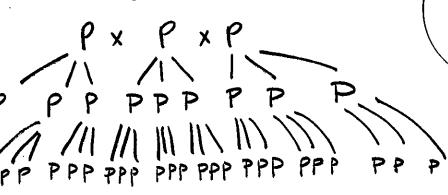
100'

They are all equal to the base value.

Example 3: Real world problems using exponents

Hadley's softball team has a phone tree in case a game is canceled. The coach calls 3 players. Then each of those players call 3 players, and so on.

How many players will be notified during the third round of calls?



3° or 27 players

Exercise 3

Tim is reading a book. On Monday he reads 2 pages. On each day after that, he reads 2 times the number of pages that he read on the previous day. How many pages does he read on Thursday?

24 or 16 pages

Problem Set

Write each expression in exponential form.

1)
$$6 \times 6 \times 6 = 216$$

3)
$$\frac{7}{9} \times \frac{7}{9} = \left(\frac{7}{9}\right)^{\frac{3}{9}}$$

4)
$$3.5 \times 3.5 = 3.5^2$$

Find the value of each expression.

6)
$$0.2^3$$

7)
$$(\frac{1}{4})^2$$
 3

10)
$$(\frac{1}{3})^3$$

Write the missing exponent.

13)	10,000 =	10#
13)	10,000 -	10 '

16)
$$\frac{1}{169} = \left(\frac{1}{13}\right)^{12}$$

Write the missing base.

19)
$$\frac{1}{9} = \left(\frac{1}{2}\right)^{2}$$

For Problems 21 and 22, write the answer with and without using an exponent.

21) One student thought two to the third power was equal to six. What mistake do you think he made, and how would you help him fix his mistake?

23 ≠ 6

An exponent means repeated multiplication of the base not just multiplication of base and exponent.

22) Some number can be written as powers of different bases. For example, $81 = 9^2$ and $81 = 3^4$. Write the number 64 using two different bases.

$$64 = 2^{6}$$
 $64 = 4^{3}$
 $64 = 9^{2}$

Evaluating Numerical Expressions Day 1

Learning Target: I can evaluate numerical expressions with exponents. (6.EE. 1)

Do Now

Exponential Form	Expanded Form	Value
. 5 ¹	5	5
52	5 x 5	25
53	5×5×5	125
54	5 x 5 x 5 x 5	625
5 ⁵	5x5x5x5x5	3.125

Opening- Exploring the Order of Operations

Evaluate the following expression. Show how you arrived at your answer.

students may provide either prot answer in with

Aultiplication and a thought of as repeated as

Multiplication can be thought of as repeated addition and division can be thought of repeated subtraction. Multiplication and division are more powerful than addition and subtraction, which led mathematicians to develop the order of operations in this way.

When we evaluate expressions that have any of these four operations, we always calculate multiplication and division before doing any addition or subtraction. Since multiplication and division are equally powerful, we simply evaluate these two operations as they are written in the expression, from left to right. Addition and subtraction are at the same level in the order of operations and are evaluated from left to right in an expression.

	First	≫ Second 7
Steps for Evaluating	Multiply or Divide × or ÷	Add or Subtract + or -
Expressions	**From left to right (whichever comes first in the expression)	From left to right (whichever comes first in the expression)

 $3 + (4 \times 2)$

The diagram correctly models the expression $3 + 4 \times 2$.



Example 1- Expressions with Only Addition, Subtraction, Multiplication, and Division

Evaluate the following expressions using the order of operations:

Multiply

Add

18

Divide Multiplu

10-4

<u>X</u>

Exercise 1

2)
$$9 \div 3 \times 2$$

3)
$$4 + 6 \times 4 \div 8$$

4)
$$3 \times 5 + 2 \times 8 + 2$$

Example 2- Expressions with Four Operations and Exponents

If exponents are present in an expression, they are evaluated before any multiplication or division.

Steps for	First	Second	Third
Evaluating Expressions With Exponents	Evaluate the exponents	Multiply or Divide × or ÷ **From left to right (whichever comes first in the	Add or Subtract + or - From left to right (whichever comes first in the

Evaluate the following expressions using the order of operations:

- 1) $4 + 9^{2} \div 3 \times 2 2$ Step 1: Exponent

 - 4+81 ÷ 3×2-2 Step 2: Divide

 - 4+27 × 2 · 2 Step 3: Multiply

 4+50-2 Step 4: Add
 - 4 + 54 2

58 - 2

Step 5: Subtract

56

- 2) $90 5^2 \times 3$

90-25x3 exponent 90-75 multiply

3) $4^3 + 2 \times 8$

Exercise 2

1)
$$5 + 18 \div 3^2 \overset{3/3}{\downarrow}$$

5+2



2)
$$21 + \frac{3^2}{3}$$



3)
$$6 \times 2^3 \div 3 + 1$$

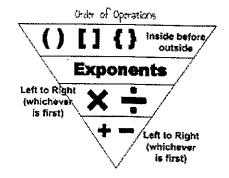


Example 3- Expressions with Parentheses

Parentheses tell us that in certain circumstances or scenarios, we need to do things out of the usual order. Operations inside grouping symbols are always evaluated first, before exponents and any operations.

When we evaluate expressions, we must agree on one set of rules so that everyone arrives at the same correct answer. You can use order of operations to simplify numerical expressions.

Order of Operations		
U P	Perform operations in parentheses.	
QE	2. Find the value of numbers with exponents.	
□ M D (L→R)	3. Multiply or divide from left to right.	
□ A S(L→R)	4. Add or subtract from left to right.	



Consider this scenario: A family of 4 goes to a soccer game. Tickets are \$5.00 each. The mom also buys a soft drink for \$2.00. How would you write this expression?

Model of this scenario	How much will this outing cost?
	(4x5)+2 20+2
(4 x5) +2	\$ 22

Consider a different scenario: The same family goes to the soccer game as before, but each of the family members wants a drink. How would you write this expression?

Model of this scenario	How much will this outing cost?
	4 x (5+2)
	4 X 7 4 2 8
How many groups are there?	Why would you add the 5 and 2 first? We must determine how much each person spends.
What does each group comprise? $5+2=7$	much each person spends.

Exercise 3

1)
$$4 \times (9 \div 3)^2$$

2)
$$8 + \frac{(12-8)^2}{2}$$

$$8 + \frac{4^2}{2}$$

3)
$$2 \times (3 + 4^2)$$



4)
$$(32 - 2^2) \div (12 - 8)$$

$$(32-4)+(4)$$

Problem Set







3)
$$2 \times 5 + 3 \times 8 + 5$$

5)
$$32 \div 4^2 + 2$$

$$\frac{27}{3} + 12$$
 $9 + 12$
 (21)

7)
$$\frac{(54 \div 6)^{-3}}{3} + 3$$

$$\frac{9^{3}}{3} + 3$$

$$\frac{729}{3} + 3 = 243 + 3 = 246$$

8)
$$4 \times (9 \div 3)^2$$

$$(9)$$
 4 × (2 + 2^4)

10)
$$(4 \times 2) + 2^4$$

13) Jay evaluated the expression
$$3 \times (3 + 12 \div 3) - 4$$
. For his first step he added $3 + 12$ to get 15. What was his error?

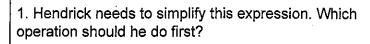
Evaluate the expression correctly.

Jay worked inside the parenthesis first, but he should have performed division not addition.

Evaluating and Comparing Numerical Expressions

Learning Target: I can evaluate and compare numerical expressions (6.EE.1, 6.EE.4)

Do Now



$$34 - 10 \div 5 \times (6 \times 2)$$

2. Evaluate the Expression:

What operation should be done first and why?

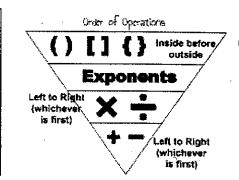
Division ble it is the "most powerful" operation inside the

parenthesis

Example 1 - Nested Parentheses

Recall that when evaluating expressions, we must follow a set of rules so that everyone arrives at the same correct answer. This is known as the **order of operations** and is used to simplify numerical expressions.

Order of Operations	
O P	Perform operations in parentheses.
C) E	2. Find the value of numbers with exponents.
☐ M D (L→R)	3. Multiply or divide from left to right.
□ A S(L→R)	4. Add or subtract from left to right.



We begin by evaluating the information INSIDE the parentheses first. If there are two or more sets of parentheses in an expression, evaluate the innermost parentheses first, and then work outward. Evaluate the following expression using the order of operations:

_ +	
÷	. =
+	
حي `	
X	

$$\frac{(6-(1+2))}{3}$$
6-3
3 = 1

Example 2 - Other Grouping Symbols

Parenthesis aren't the only symbols used to group values in an expression. Expressions using different types of grouping symbols and are solved just like nesting parentheses from Example 1. **Evaluate the innermost set of grouping symbols FIRST** and work your way outward and if the parenthesis are not nest, work from left to right.

Parenthesis (1) Bra	
Parenthesis () Rra	ckets* [] Braces {-}
	Barrier
THE RESIDENCE OF THE PROPERTY	

Evaluate the following expression:

Exercise 2

$$\begin{cases}
40+7 \\
40+7 \\
47+25
\end{cases}$$

$$\frac{\frac{(6+4^{2})}{[(3\times3)\times4]}}{\frac{(6+11a)}{[9\times4]}}$$

$$\frac{22}{36} = 18$$

Example 3- Comparing Numerical Expressions

To compare numerical expressions, evaluate both sides of the equation. Then compare the values using inequality symbols (<, >, =).

$$8 \times 7 - 6^{2} + 3$$
 $3 \times (12 - 5) + 8 \times 4$ $3 \times (7 - 6^{2} + 3)$ $3 \times (7 + 8 \times 4)$ $3 \times 7 + 8 \times 4$ $3 \times 7 + 8$

Exercise 3

Problem Set

Evaluate the following expressions

(32 -
$$2^2$$
) ÷ (8 - 4)
(32 - 4) ÷ 4
28 ÷ 4

2)
$$[40 - (16 + 12)] + (4 + 2 \times 3)^{2}$$

 $[40 - 28] + (2 \times 3)^{2}$
 $|2 + 6^{2}$
 $|2 + 36$
 $|48$

3)
$$\frac{((4+4)+3)^{2}}{11} + 7 \times 2$$

$$\frac{(8+3)^{2}}{11} + 14$$

$$\frac{11^{2}}{11} + 14$$

$$1\frac{12^{1}}{11}+14$$
 $11+14$
 25

 $12 \div 3 \times 3^2$ \bigcirc $12 \times 3 \div 3^2$

5)
$$\frac{5^2 - 4 \cdot 3 \div 12}{12 - 2 \div 14}$$
25 - $4 \cdot 3 \div 12$
24
25 - $12 \div 12$

$$\frac{25-1}{24}$$

$$\frac{24}{24} = 1$$

6)
$$39 \div (2+1) - 2 \times (4+1)$$

 $39 \div 3 - 2 \times 5$
 $13 - 10$

12:3×9

4)

1)
$$4^{3}$$
 $\cancel{\bigcirc}$ 2^{5} $\cancel{\bigcirc}$ $\cancel{$

12×3÷9

Writing Equivalent Expressions

Learning Target: I can compare numerical expressions and I can write equivalent expressions. (6.EE.3, 6.EE.4)

Do Now - Evaluate the Expression

$$7^{3} + \{7^{2} + [22 + (9 + 7^{2})]\}$$

$$7^{3} + \{7^{2} + [22 + (9 + 49)]\}$$

$$343 + \{49 + (22 + 58]\}$$

$$\{49 + 80\}$$

$$343 + 129$$

$$472$$

Example 1- Which Numerical Expression Matches this Scenario?

- 1) John makes flower arrangements as gifts. For one bouquet, he places 2 red and 3 white roses together. Three of his friends each buy two bouquets. How many flowers does John sell in all?

- 2) Tim is reading a book. On Monday, he reads 3 pages. On each day after that, he reads three times the number that he read the previous day. How many pages does he read on Thursday?
 - $\begin{array}{c}
 & 3+3+3+3 \\
 & 3^{3} \\
 & 3\times3\times3\times3\times3\times3
 \end{array}$
- 3) Start with 3 and add 2. Then double that amount. Then double it again.
 - 3+2x2x2
 - $(3 + 2)^2$
 - $3 + 2 \times (2 \times 2)$

• $(3+2) \times 2 \times 2$

4) You have \$5 and spend \$3, then earn \$2 the next day.

- (5 3) 2
- 5 (3 + 2)
- 5+2-3

Exercise 1

- 5) You mix some chemicals for a crystal garden and let it start growing at noon. At 1:00 pm the crystal weighs 2 grams. At 2:00 pm, its weight doubles: At 3:00 pm, you notice its weight doubles again. What will it probably weigh at 5:00 pm?
 - 2+2+2+2+2
 - 2x2x2x2
 - 2⁴ • 2⁵

Example 2- Writing Numerical Expressions

Sometimes in math we describe an expression with a phrase.

For example: the phrase
"2 more than 5":

* can be written as the expression
2+5

When writing a numerical expression that represents the phrase, look for the key words and phrases that signal the different math operations. You must remember the order of operations when you write a numerical expression.

	Common Common	Keywords	
Addition	Subtraction	Multiplication	Division
plus sum more than increased by	Subtracted minus difference less than decreased by	times product	divide quotient

Phrase	Numerical Expression
The sum of 4 and 2 divided by 3	(4+2)÷3
The difference of 9 and 5, doubled	(9-5) x 2
Six more than the product of 2 and 7	(2x7)+6

2 more than 7	81 divided by 9
2+7	81 ÷ 9
8 less than the <u>product</u> of 7 and 2 (7 * 2) - 8	the sum of 6 and the product of 3 and 4 $(Q + (3 * 4))$
10 times the difference of 9 and 4	The quotient of 60 and 5 minus the sum of 7 and 2 $(20 \div 5) - (7 + 2)$

Example 3 - Writing Numerical Expression from a story problem

1. You plant a tree that is 10 inches tall. The height increases by 15 inches each year. Write and simplify a numerical expression to find the height after 9 years.

2. Your sister has 8 game tokens. Your friend has 5 more than twice as many game tokens as your sister. Write and simplify a numerical expression for the number of game tokens your friend has.

1. Daniel makes 100 dollars each week. He worked for 4 weeks this summer. How much did Daniel make in all?

2. Susie ran a race. She ran 5 miles an hour. The race took her 2 hours to complete. How long was the race?

3. The Aces card team won \$538 playing poker in a tournament last summer. They entered into another tournament the following week and won an additional \$125. The total winnings were split evening among the 4 players. How much did each team player recieve?

Problem Set

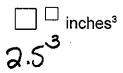
Write and simplify a numerical expression.

7) 36 divided by the product of 6 and 3

8) The sum of 4 and 8 minus 7

9) The nature park has a pride of 7 adult lions and 4 cubs. The adults eat 6 pounds of meat each day and the cubs eat 3 pounds. Write and simplify a numerical expression to find the amount of meat consumed each day by the lions.

16) The formula for the volume of a cube is $1 \times w \times h$, but since all sides are the same length in a cube, we
can write the volume of a cube as s^3 where s is the length of one side. Write an expression for the volume of a cube that has a side length of 2.5 inches.



17) Jack reads in a science book that germs or bacteria doubles every 2 minutes. If there are 1,000 bacteria on your finger tip, how many bacteria will there be after 10 minutes if you do not wash your hands.

$$1,600 \times 2 \times 2 \times 2 \times 2 \times 2 = 1,000 \times 2 = 5$$

- 18) Ellen is playing a video game in which she captures butterflies. There are 3 butterflies on the screen, but the number of butterflies doubles every minute. After 4 minutes, she was able to capture 7 of the butterflies.
 - A) Write an expression for the number of butterflies after 4 minutes. Use a power of 2 in your answer.

B) Write an expression for the number of butterflies remaining after Ellen captured the 7 butterflies. Simplify the expression. $3 \times 2^4 - 7$

- 19) What is the largest number you can make with the digits 1, 2, 3, 4 using <u>any</u> operation (parenthesis, exponent, +, -, \times , \div).
- **You must operate on each number separately (that is you cannot just combine the digits like 4,321).